The Integral Definition of Magnetic Vector Potential

Recall for **electrostatics**, we began with the definition of **electric scalar potential**:

$$\mathbf{E}(\bar{r}) = -\nabla V(\bar{r})$$

And then taking a **contour** integral of each side we discovered:

$$\int_C \mathbf{E}(\bar{r}) \cdot \overline{d\ell} = -\int_C \nabla V(\bar{r}) \cdot \overline{d\ell}$$

$$\int_C \mathbf{E}(\bar{r}) \cdot \overline{d\ell} = V(\bar{r}_a) - V(\bar{r}_b)$$

We can perform an **analogous** procedure for magnetic vector potential! Recall magnetic flux density $\mathbf{B}(\bar{r})$ can be written in terms of the magnetic vector potential $\mathbf{A}(\bar{r})$:

$$\mathbf{B}(\bar{r}) = \nabla \times \mathbf{A}(\bar{r})$$

Say we **integrate** both sides over some **surface** S:

$$\iint_S \mathbf{B}(\bar{r}) \cdot \overline{ds} = \iint_S \nabla \times \mathbf{A}(\bar{r}) \cdot \overline{ds}$$

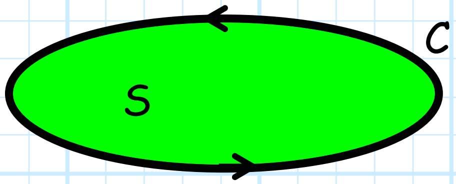
We can apply **Stoke's theorem** to write the right side as:

$$\iint_S \nabla \times \mathbf{A}(\bar{r}) \cdot \overline{ds} = \oint_C \mathbf{A}(\bar{r}) \cdot \overline{d\ell}$$

Therefore, we find that we can also define magnetic vector potential in an **integral form** as:

$$\iint_S \mathbf{B}(\bar{r}) \cdot \overline{ds} = \oint_C \mathbf{A}(\bar{r}) \cdot \overline{d\ell}$$

where contour *C* defines the **border** of surface *S*.



Consider now the **meaning** of the integral:

$$\iint_S \mathbf{B}(\bar{r}) \cdot \overline{ds}$$

This integral is remarkably **similar** to:

$$\iint_S \mathbf{J}(\bar{r}) \cdot \overline{ds}$$

where:

$$\mathbf{B}(\bar{r}) \doteq \text{magnetic flux } \textbf{density} \quad \left[\frac{Webers}{meters^2}\right]$$

and:

$$\mathbf{J}(\bar{r}) \doteq \text{current } \textbf{density} \quad \left[\frac{Amperes}{meters^2}\right]$$

Recall that integrating the **current density** (in *amps/m^2*) over some surface S (in m^2), provided us the **total current** I flowing through surface S:

$$\iint_S \mathbf{J}(\bar{r}) \cdot \overline{ds} = I$$

Similarly, integrating the **magnetic flux density** (in *webers/m^2*) over some surface S (in m^2), provided us the **total magnetic flux** Φ flowing through surface S:

$$\iint_S \mathbf{B}(\bar{r}) \cdot \overline{ds} = \Phi$$

where Φ is defined as:

$$\Phi \doteq \text{magnetic flux} \quad [Webers]$$

Using the equations derived previously, we can **directly** relate magnetic vector potential $\mathbf{A}(\bar{r})$ to magnetic flux as:

$$\Phi = \oint_C \mathbf{A}(\bar{r}) \cdot \overline{d\ell}$$

where we recall that the **units** for magnetic vector potential are *Webers/m.*

Note the similarities of the above expression to the integral form of **Ampere's Law!**

$$I = \frac{1}{\mu_0} \oint_C \mathbf{B}(\bar{r}) \cdot \overline{d\ell}$$